DOI: https://doi.org/10.33216/2222-3428-2021-21-12

UDC 629.4.014.22

Goolak S., Sapronova S., Tkachenko V., Riabov Ie., Overianova L., Yeritsyan B.

MATHEMATICAL MODEL OF MECHANICAL SUBSYSTEM OF TRACTION ELECTRIC DRIVE OF AN ELECTRIC LOCOMOTIVE

The mathematical model of the mechanical subsystem of the traction electric drive of the four - axle section of the locomotive is developed in the article. The peculiarity of the model is taking into account the interconnected vertical oscillations of the locomotive and torsional oscillations in traction transmission and the influence of the anti-discharge device on the processes in the traction electric drive. This allows taking into account the relationship of oscillations of the crew of the locomotive, which have a significant impact on the processes in the electrical part of the traction drive. The model describes the oscillations of the locomotive body, two carts, four wheel pairs, traction motors and gearboxes. When developing the model, the assumption of linear dependence of stiffness and damping coefficients on displacements and velocities is accepted. Coupling of a wheel with a rail is described by means of the approximated dependence. The oscillations of the rails and the bases of the rail track are introduced into the model. As perturbation for vertical oscillations the roughness of a rail track is accepted. The model takes into account the influence of traction force and vertical oscillations of the crew on the unloading of the axles of the locomotive. This will allow us to study the processes of realization of the locomotive's maximum traction forces in the traction mode and in the mode of electrodynamics braking.

The use of the developed mathematical model of the mechanical subsystem will allow taking more fully into account the mutual influence of electrical processes and mechanical oscillations on each other, which will increase the accuracy of modeling. The model can be used for research of freight electric locomotives such as VL10, VL11, VL82, VL80, 2EL4, 2EL5 in order to further improve their traction electric drives, in particular, to determine the rational modes of application of the anti-unloading device in traction and braking modes. Further application of the mathematical model is possible to assess the performance of the traction drive and the locomotive in general in the study of modern traction drives, in particular, asynchronous, the use of which is possible on the above locomotives.

Keywords: *electric locomotive, traction electric drive, bogie, oscillation*

Introduction. The National Economic Strategy for the period up to 2030 [1] provides for the renewal and modernization of the rolling stock of JSC Ukrzaliznytsia. The latter will be most effective if, as a result of modernization, the key technical characteristics of the rolling stock are brought closer to the level of modern models. Taking into account the work on the electrification of the railways of Ukraine, it seems advisable to improve the main-line electric freight locomotives of the VL10, VL11, VL80, VL82 types, as well as the 2EL5K and 2ES5K electric locomotives in order to increase their traction and energy performance.

Prediction of the characteristics of modernized electric locomotives can be achieved through mathematical modeling, which allows one to study processes in various systems and predict the properties of an electric locomotive as a whole. To carry out such work, appropriate mathematical models are required that reliably describe the processes in the systems under study.

The modern approach to modeling processes in such a complex electromechatronic system as a traction electric drive of a locomotive is to develop a complex mathematical model [2, 3] using specialized software that allows modeling with a high degree of detail of the studied locomotive systems [4-6]. However, for problems where electrical processes in a traction electric drive and its components are studied as a priority, it is sufficient to use simplified, less detailed mathematical models of the mechanical subsystem. For example, in [7, 8], the mechanical subsystem is described in the form of equations of torsional vibrations in the traction drive of a locomotive. At the same time, it was shown in [9] that torsional vibrations are interconnected with vertical vibrations of the locomotive undercarriage assembly, therefore, models that take this circumstance into account will be more reliable. Such models are used in [10, 11]. In this case, the assumption is made that horizontal and vertical oscillations are independent of each other [9, 12]. Models of this type - taking into account the vertical vibrations of the locomotive and torsional vibrations of the traction drive - seem to be more suitable for tasks in which the main research is directed to the processes in the electrical subsystem, but there is a significant mutual influence of the electrical and mechanical subsystems of the traction electric drive.

A mathematical model of the mechanical subsystem of an electric traction drive of an electric locomotive can be developed using the recommendations that are used to study the dynamics of a locomotive and assess its driving performance [13-15]. This model is supplemented by a description of the traction drive torsional vibrations, containing the equations of the traction motor and gearbox vibrations. External forces will be traction and braking forces (with

electrodynamic braking), resistance forces to motion, as well as axle loading forces during the operation of counter-unloading devices, which are equipped with the considered electric locomotives. It should be noted that there are practically no studies concerning the effect of anti-unloading devices on dynamic processes in the mechanical subsystem and the traction electric drive as a whole. It also seems necessary to take into account the redistribution of axle loads that occurs when traction (braking) forces are realized.

Thus, the **aim** of the article is to develop a mathematical model of the mechanical subsystem of the traction electric drive of the four-axle section of an electric locomotive, taking into account the vertical vibrations of the locomotive and torsional vibrations in the traction drive of the four-axle section, as well as the operation of the anti-unloading device.

Research results. A mathematical model is being developed for a four-axle section of a mainline freight electric locomotive. A general description of the design of electric locomotives can be found in the relevant literature ([16-18], etc.), here we only note the design features that affect the development of a mathematical model. The electric locomotives are equipped with a first-class traction drive, in which the traction motor and the traction gearbox have axial support. The traction motor is supported on the axle of the wheelset through plain bearings, on the frame through a rubber-metal support. The gear train is double-sided (spaced chevron), there are no elastic elements in the gear wheel design. The wheel-motor unit is connected to the bogie frame through leashes with rubber-metal bushings. The transfer of traction forces from the bogie to the body is carried out through a ball connection, which does not allow the body and bogies to move along the longitudinal axis of the electric locomotive. A distinctive feature of electric locomotives is the use of anti-unloading devices to increase the use of the adhesion weight of the locomotive: anti-unloading devices provide additional loading of the extreme axes of the electric locomotive section when traction forces are applied (the first wheelset is loaded) and electrodynamic braking (the last wheelset is loaded along the way).

When developing the model, the following assumptions were made [12, 13]:

1) the body, bogies and unspring elements of the locomotive undercarriage assembly are considered absolutely rigid bodies;

2) the movement of wheelsets on the rails is uninterrupted;

3) the upper structure of the track is presented in the form of a concentrated mass with elastic and dissipative parameters;

4) the crew is symmetrical, there are no technological deviations and clearances;

5) fluctuations are small;

6) the disturbing effect on the crew is assumed to be random;

7) is considered freight traffic at speeds of no more than 100 km/h in straight horizontal sections of the track.

We also neglect the elastic-dissipative properties of the wheelset axle [9, 19], which makes it possible to exclude a separate mathematical description of the movement of the left and right wheels. We assume that the spring suspension and vibration dampers of the body stage are installed in a vertical transverse plane passing through the center of the ball connection of the body and the bogie, and are replaced by devices with equivalent parameters. We also assume that the spring suspension and vibration dampers of the axle-box stage are installed in vertical and horizontal planes passing through the axis of rotation of the wheelset and are replaced by devices with equivalent parameters. The counter-unloading device is modeled by an elastic-dissipative element with constant stiffness and damping coefficients.

Figure 1 shows the computational model of the section of the investigated electric locomotive.

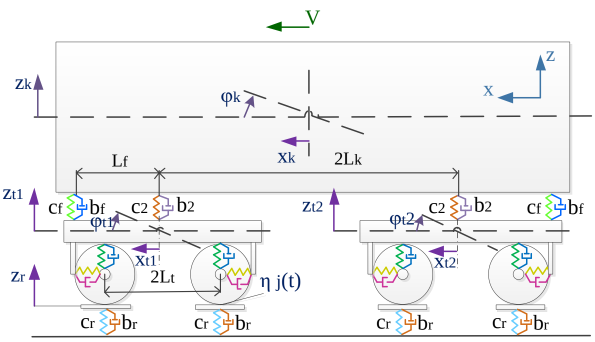


Figure 1 - Design model of a four-axle section of an electric locomotive
($2L_k$ – body base, $2L_t$ – bogie base, L_f – distance from the pivot assembly to the end beam of the bogie)

The design model of the wheel-motor unit was adopted in accordance with the recommendations given in [9, 14, 19, 20]. The elasticity of the gearing of the traction reducer has been added to the model. The design model diagram is shown in Fig. 2.

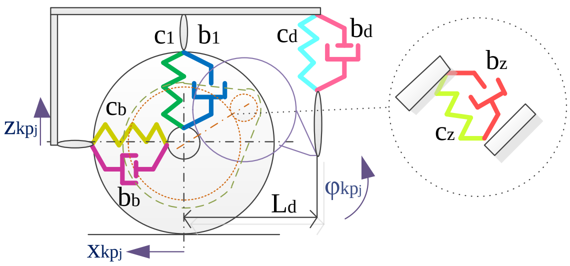


Figure 2 - Design model of the wheel-motor unit

Differential equations of oscillations are compiled using the Lagrange equations of the second kind

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial \Phi}{\partial \dot{q}_i}+\frac{\partial \Pi}{\partial q_i}=Q_i\,, \tag{1}$$

where T – kinetic energy;

Π – potential energy;

Φ – dissipative function;

q_i – generalized coordinate;

$\dot{q}_i$ – generalized speed;

Q_i – external force acting along the coordinate.

The kinetic energy is calculated by the expression

$$T=\frac{1}{2}\left(M_k\dot{x}_k^{\,2}+M_k\dot{z}_k^{\,2}+J_k\dot{\varphi}_k^{\,2}\right)+\frac{1}{2}\sum_{i=1}^{2}\left(M_t\dot{x}_{ti}^{\,2}+M_t\dot{z}_{ti}^{\,2}+J_t\dot{\varphi}_{ti}^{\,2}\right)+$$
$$+\frac{1}{2}\sum_{j=1}^{4}\left(M_{kp}\dot{x}_{kpj}^{\,2}+M_{kp}\dot{z}_{kpj}^{\,2}+J_{kp}\dot{\varphi}_{kpj}^{\,2}\right)+\frac{1}{2}\sum_{j=1}^{4}\left(M_d\dot{x}_{dj}^{\,2}+M_d\dot{z}_{dj}^{\,2}+J_d\dot{\varphi}_{dj}^{\,2}\right)+\frac{1}{2}\sum_{j=1}^{4}J_a\dot{\varphi}_{aj}^{\,2}+\frac{1}{2}\sum_{j=1}^{4}M_r\dot{z}_{rj}^{\,2}, \tag{2}$$

where M_k – body weight,

J_k – body moment of inertia,

M_t – bogie frame weight,

J_t – moment of inertia of the bogie frame,

M_{kp} – weight of the wheelset (with axle boxes and toothed wheels of traction gearbox),

J_{kp} – moment of inertia of a wheel pair (with gear wheels of traction gearbox),

M_d – the mass of the traction motor (taking into account the mass of the casings and gears of the traction gearbox),

J_d – moment of inertia of the traction motor (taking into account the casings, gears of the traction reducers and the reduced moment of inertia of the armature) relative to the axis of the wheelset,

J_a – moment of inertia of the armature (taking into account the gears of the traction reducers) relative to the axis of rotation of the electric motor,

M_r – reduced mass of the rail-sleeper grate and track base (per wheelset),

$\dot{x}_k, \dot{z}_k, \dot{\varphi}_k$ – speed of the center of mass of the body in the longitudinal and vertical directions and the speed of rotation of the body when galloping, respectively,

$\dot{x}_{ti}, \dot{z}_{ti}, \dot{\varphi}_{ti}$ – the speed of the center of mass of the bogie frame in the longitudinal and vertical directions and the speed of rotation of the frame when galloping the bogie, respectively ($i=1,2$),

$\dot{x}_{kpj}, \dot{z}_{kpj}, \dot{\varphi}_{kpj}$ – speed of the center of mass of the wheelset in the longitudinal and vertical directions and the angular speed of the wheelset, respectively ($j=\overline{1,4}$),

$\dot{x}_{dj}, \dot{z}_{dj}, \dot{\varphi}_{dj}$ – speed of the center of mass of the electric motor in the longitudinal and vertical directions and the speed of rotation of the electric motor relative to the axis of the wheelset, respectively,

$\dot{\varphi}_{aj}$ – motor armature rotation speed,

$\dot{z}_{rj}$ – vertical speed of the rail-sleeper grid.

The horizontal speed of the bogies is expressed in terms of the speed of the body

$$\dot{x}_{ti}=\dot{x}_k\pm\dot{\varphi}_k H_k\,, \tag{3}$$

where H_k – the distance from the center of the ball link on the bogie to the center of mass of the body. The second term takes into account the twitching of the cart when the body is galloping.

Horizontal speed of the center of mass of the traction motor (neglecting the clearances in the motor-axial bearings)

$$\dot{x}_{dj} = \dot{x}_{kpj} . \tag{4}$$

The vertical speed of the center of mass of the traction motor [21]

$$\dot{z}_{dj} = \frac{\dot{z}_{ti} L_1 + \dot{z}_{kpj} L_2}{L_d} , \tag{5}$$

where $L_1, L_2, L_d = L_1 + L_2$ – horizontal distance from the center of mass of the electric motor to the point of its attachment to the bogie frame, the axle of the wheelset, respectively, and the total length of the engine mount. In the calculations, we assume that the fastening of the electric motor to the bogie frame is located in a plane passing through the center of the pivot assembly.

Speed of rotation of the traction motor frame around the axis of the wheelset [21]

$$\dot{\varphi}_{dj} = \frac{\dot{z}_{kpj} - \dot{z}_{ti}}{L_d} . \tag{6}$$

Since the rolling of the wheelset on the rail is continuous, the vertical speed of the track element under the wheelset is expressed as follows

$$\dot{z}_{rj} = \dot{z}_{kpj} - \dot{\eta}_j , \tag{7}$$

where $\dot{\eta}_j = \dot{\eta}(x_{kpj})$ – time derivative of the unevenness of the track (x_{kpj} – longitudinal displacement of the wheelset).

The potential energy of the system is calculated by the expression

$$\begin{gathered} \Pi = \frac{1}{2}\sum_{i=1}^{2} c_2 \left(\Delta z_{2i}\right)^2 + \frac{1}{2}\sum_{i=1}^{2} k_{fi} c_f \left(\Delta z_{fi}\right)^2 + \frac{1}{2}\sum_{j=1}^{4} c_1 \left(\Delta z_{1j}\right)^2 + \frac{1}{2}\sum_{j=1}^{4} c_b \left(\Delta x_{kpj}\right)^2 + \\ + \frac{1}{2}\sum_{j=1}^{4} c_{b\varphi} \left(\Delta \varphi_{b\varphi j}\right)^2 + \frac{1}{2}\sum_{j=1}^{4} c_d \left(\Delta z_{dj}\right)^2 + \frac{1}{2}\sum_{j=1}^{4} c_z \left(\Delta \varphi_{zj}\right)^2 + \frac{1}{2}\sum_{j=1}^{4} c_r \left(\Delta z_{rj}\right)^2 \end{gathered} , \tag{8}$$

where c_2 – coefficient of rigidity of the spring suspension of the body stage (equivalent),

c_f – coefficient of rigidity of the anti-unloading device,

c_1 – coefficient of rigidity of the spring suspension of the axle-box stage (equivalent),

c_b – coefficient of rigidity of the leaders (equivalent),

$c_{b\varphi}$ – coefficient of torsional rigidity of the leaders (equivalent),

c_d – coefficient of rigidity of electric motor suspension,

c_z – coefficient of rigidity of the gearing of the traction gearbox (equivalent),

c_r – coefficient of rigidity of the rail-sleeper lattice and the track base,

Δz_{2i} – movement of the spring suspension of the body stage,

Δz_{fi} – movement of the support of the anti-unloading device,

Δz_{1j} – vertical movement of the spring suspension in the axle-box stage,

Δx_{kpj} – horizontal movement of the rubber elements of the leashes,

$\Delta \varphi_{b\varphi j}$ – the angle of twisting of the rubber elements of the leashes,

Δz_{dj} – deformation of the rubber support of the traction motor,

$\Delta \varphi_{zj}$ – deformation in the gearing of the traction gearbox,

Δz_{rj} – deformation of the rail-sleeper grid,

k_{fi} – boolean variable. If the anti-unloading device is on, then $k_{fi} = 1$ otherwise $k_{fi} = 0$. In this case, the simultaneous switching on of the anti-unloading device of the front and rear bogies is not allowed.

The movement of the spring suspension of the body stage is calculated by the expression (neglect the galloping of the bogie)

$$\Delta z_{2i} = (z_k \pm \varphi_k L_k) - z_{ti} , \tag{9}$$

where φ_k is the pitching angle of the locomotive body,

z_{ti} – vertical movement of the bogie frame,

L_k – half the distance between the centers of the ball joints on the body.

The "+" sign is taken for the first trolley in the direction of travel, the "–" – for the second trolley in the direction of movement.

Vertical movement of the counter-unloading device hinge

$$\Delta z_{fi} = (z_k \pm \varphi_k (L_k + L_f)) - (z_{ti} \pm \varphi_{ti} L_f), \tag{10}$$

where φ_{ti} – bogie galloping angle,

L_f – the distance from the center of the ball joint to the lining for supporting the anti-unloading device roller on the end beam of the bogie.

Signs "+" are taken for the first bogie in the direction of travel, signs "–" – for the second bogie in the direction of travel.

Vertical movement of the spring suspension in the axle-box step

$$\Delta z_{1j} = (z_{ti} \pm \varphi_{ti} L_t) - z_{kpj}, \tag{11}$$

where z_{kpj} is the vertical movement of the wheelset. The "+" sign is taken for the first wheelset in the direction of travel for each bogie, the "–" sign – for the second wheelset in the direction of travel of each bogie.

Horizontal deformation of the leader rubber elements

$$\Delta x_{kpj} = x_{kpj} - \left(x_{ti} \pm \varphi_{ti} H_t\right) = x_{kpj} - \left(x_k \pm \varphi_k H_k \pm \varphi_{ti} H_t\right), \tag{12}$$

where H_t – distance from the axle of the wheelset to the center of mass of the bogie. The "+" sign in front of the term $\varphi_k H_k$ is for the first and second wheelsets, the "-" sign in front of the term $\varphi_k H_k$ is for the third and fourth wheelsets. The "+" sign in front of the term $\varphi_{ti} H_t$ is taken for the first and third wheelset, the "–" sign in front of the term $\varphi_{ti} H_t$ is taken for the second and fourth wheelset.

The angle of twisting of the rubber elements of the leaders is calculated by the expression

$$\Delta\varphi_{b\varphi j} = \frac{z_{ti} - z_{kpj}}{L_b} \tag{13}$$

where L_b – the distance between the axes of the rubber elements in the leader.

Vertical deformation of the traction motor support (neglect the galloping of the bogie)

$$\Delta z_{dj} = z_{ti} - z_{kpj} \tag{14}$$

Deformation in gearing

$$\Delta\varphi_{zj} = R_2 \varphi_{kpj} - R_1 \varphi_{aj}, \tag{15}$$

where $\varphi_{kpj}, \varphi_{aj}$ – the angle of rotation of the *j*-th wheelset and the armature of the *j*-th electric motor, respectively R_1, R_2 – radii of the gear and toothed wheel of the traction gearbox.

Deformation of the rail-sleeper lattice

$$\Delta z_{rj} = z_{kpj} - \eta_j \tag{16}$$

Here $\eta_j = \eta(x_{kpj})$ is the unevenness of the path.

The dissipative function is calculated by the expression

$$\begin{aligned}\Phi = &\frac{1}{2}\sum_{i=1}^{2} b_2 \left(\dot{\Delta} z_{2i}\right)^2 + \frac{1}{2}\sum_{i=1}^{2} k_{fi} b_f \left(\dot{\Delta} z_{fi}\right)^2 + \frac{1}{2}\sum_{j=1}^{4} b_1 \left(\dot{\Delta} z_{1j}\right)^2 + \frac{1}{2}\sum_{j=1}^{4} b_b \left(\dot{\Delta} x_{kpj}\right)^2 + \\ &+ \frac{1}{2}\sum_{j=1}^{4} b_{b\varphi} \left(\dot{\Delta}\varphi_{b\varphi j}\right)^2 + \frac{1}{2}\sum_{j=1}^{4} b_d \left(\dot{\Delta} z_{dj}\right)^2 + + \frac{1}{2}\sum_{j=1}^{4} b_z \left(\dot{\Delta}\varphi_{zj}\right)^2 + \frac{1}{2}\sum_{j=1}^{4} b_r \left(\dot{\Delta} z_{rj}\right)^2\end{aligned} \tag{17}$$

where b_2 – damping coefficient of the spring suspension of the body stage (equivalent);

b_f – damping coefficient of the anti-unloading device (equivalent);

b_1 – damping coefficient of the axle-box spring suspension (equivalent);

b_b – compression damping coefficient of the leader;

$b_{b\varphi}$ – torsional damping coefficient of the leader;

b_d – damping coefficient of the traction motor suspension;

b_z – damping coefficient of the gearing of the traction reducer (equivalent);

b_r – damping coefficient of the rail-sleeper lattice.

In expression (17), it is assumed that the speed is calculated as $\dot{\Delta} = \frac{d}{dt}\Delta$. Then using expressions (9) – (16) we obtain

$$\dot{\Delta} z_{2i} = (\dot{z}_k \pm \dot{\varphi}_k L_k) - \dot{z}_{ti} \tag{18}$$

$$\dot{\Delta} z_{fi} = (\dot{z}_k \pm \dot{\varphi}_k (L_k + L_f)) - (\dot{z}_{ti} \pm \dot{\varphi}_{ti} L_f) \tag{19}$$

$$\dot{\Delta} z_{1j} = (\dot{z}_{ti} \pm \dot{\varphi}_{ti} L_t) - \dot{z}_{kpj} . \quad (20)$$

$$\dot{\Delta} x_{kpj} = \dot{x}_{kpj} - \left(\dot{x}_k \pm \dot{\varphi}_k H_k \pm \dot{\varphi}_{ti} H_t \right) . \quad (21)$$

$$\dot{\Delta} \varphi_{b\varphi j} = \frac{\dot{z}_{ti} - \dot{z}_{kpj}}{L_b} . \quad (22)$$

$$\dot{\Delta} z_{dj} = \left(\dot{z}_{ti} \pm \dot{\varphi}_{ti} L_t \right) - \dot{z}_{kpj} . \quad (23)$$

$$\dot{\Delta} \varphi_{zj} = R_2 \dot{\varphi}_{kpj} - R_1 \dot{\varphi}_{aj} . \quad (24)$$

$$\dot{\Delta} z_{rj} = \dot{z}_{kpj} - \dot{\eta}_j . \quad (25)$$

To calculate the generalized forces, we introduce the generalized coordinates:

– longitudinal horizontal and vertical movements and pitching angle of the body x_k, z_k, φ_k,

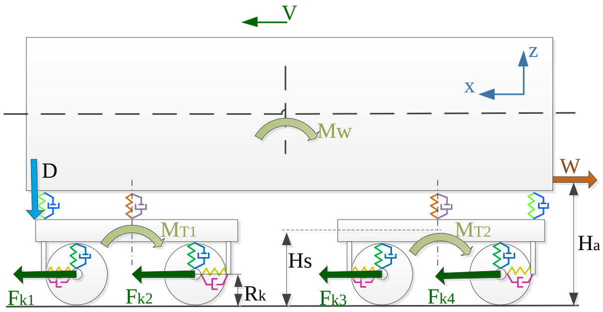
– vertical movements and angles of galloping of frame of bogies z_{ti}, φ_{ti},

– horizontal and vertical displacements and angles of rotation of wheelsets $x_{kpj}, z_{kpj}, \varphi_{kpj}$,

– angle of rotation of the traction motor armature φ_{aj}.

Thus, the system will have 23 degrees of freedom.

We introduce generalized forces as follows (Fig. 3). Consider the traction operation mode.



(

Figure 3 - Scheme of application of generalized forces and moments

(H_a – coupling height, H_s – height of the pivot assembly, R_{kp} – wheel radius)

The traction forces act along the coordinate x_k

$$F_A = \sum_{j=1}^{4} F_{kpj} = \frac{1}{R_{kp}} \sum_{j=1}^{4} M_{kpj} \quad (26)$$

where M_{kpj} – coupling moment of the j-th wheelset and the forse of the main resistance to movement W, which we neglect

On the coordinate φ_k, the moment M_W from the traction force acts on the resistance coupler

$$M_W = F_A \left(H_a - H_s \right) , \quad (27)$$

where H_a and H_s are the automatic couplings, the height and distance from the level of the rail head to the center of the bogie ball connection. We accept these values as unchanged when the locomotive vibrates.

By the coordinates z_{t1} and z_{t2} the forces act D_1 and D_2, accordingly, from the anti-unloading device: $D_i = k_{fi} \left(D_{0i} - c_{fd} \Delta f_i \right)$ – the additional loading force from the anti-unloading device, here D_{0i} – the initial force, Δf_i – the piston stroke of the cylinder of the anti-unloading device of the *i*-th bogie, c_{fd} – coefficient of rigidity.

On the coordinate φ_{t1}, the moment from the force of the anti-unloading device and the traction forces of the first bogie act

$$M_{t1} = -D_1 L_f + (F_{kp1} + F_{kp2}) \left(H_s - R_{kp} \right) \quad (28)$$

The moment from the traction force of the second bogie acts along the coordinate φ_{t2}

$$M_{t2} = D_2 L_f + (F_{kp3} + F_{kp4}) \left(H_s - R_{kp} \right) \quad (29)$$

The moments M_{aj} of the electric motors act on the coordinates φ_{aj}.

The adhesion moments M_{kpj} act on the coordinates φ_{kpj}.

The adhesion moment of the *j*-th wheelset is calculated taking into account the recommendations [9, 19, 23-26]

$$M_{kpj} = R_{kp} \Psi_0 K_{kpj} \left(N_0 + \Delta N_{kpj} \pm \Delta N_{Mkpj} \right) \quad (30)$$

where $K_{kpj} = K(u_{kpj})$ – dependence of the coefficient of adhesion on the sliding speed for the *j*-th wheelset,

Ψ_0 – potential adhesion coefficient (determined by [22]),

N_0 – static component of the normal reaction of the rails to the wheelset, equal to 235 kN,

ΔN_{kpj} – the dynamic component of the normal reaction of the rails to the wheelset,

ΔN_{Mkpj} – change in the load on the wheelset when the traction (braking) force is realized.

The sliding speed of the j-th wheelset is calculated by the expression

$$u_{kpj} = R_{kp}\dot{\varphi}_{kpj} - \dot{x}_{kpj} \tag{31}$$

Coefficient $K(u_{kpj})$ is approximated by the following formulas depending on the relative sliding speed $\frac{u}{\hat{V}_E}$ ($\hat{V}_E$ – speed expressed in m/s ($\hat{V}_E = \dot{x}$); V_E – speed expressed in km/h ($V_E = 3{,}6\hat{V}_E$)):

if $0 < u \le 0{,}14\%\hat{V}_E$

$$K = 1491{,}25\frac{u}{V_E}, \tag{32}$$

if $0{,}14\%\hat{V}_E < u \le 1{,}14\%\hat{V}_E$

$$K = \frac{381{,}6u - V_E}{360u - 0{,}04V_E}, \tag{33}$$

if $1{,}14\%\hat{V}_E < u \le 2{,}5\%\hat{V}_E$

$$K = 1{,}06 - 16{,}36\frac{u}{V_E}, \tag{34}$$

if $u \ge 2{,}5\%\hat{V}_E$

$$K = 0{,}57e^{-0{,}68(u-u_{kr})} + 0{,}36e^{-0{,}0036(u-u_{kr})} + 0{,}02e^{-1{,}5(u-u_{kr})}, \tag{35}$$

where $u_{kr} \cong 2{,}5\%\hat{V}_E$.

The dynamic component of the normal reaction of the rails to the wheelset is calculated by the expression [9]

$$\Delta N_{kpj} = \frac{M_r\left(b_1\Delta\dot{z}_{1j} + c_1\Delta z_{1j}\right) - M_{kp}\left(b_r\Delta\dot{z}_{rj} + c_r\Delta z_{rj}\right)}{M_{kp} + M_r}. \tag{36}$$

Change in the load on the rails when the wheelset implements the traction force [14]

$$\Delta N_{Mkpj} = \frac{F_{kpj}R_{kp}}{L_t}. \tag{37}$$

In this case, unloading takes place if the traction motor is located behind the wheelset in the direction of travel, and additional loading takes place if the traction motor is located in front of the wheelset.

After performing calculations according to expressions (1) - (25), we obtain 23 differential equations describing the longitudinal, vertical and angular vibrations of the locomotive and torsional vibrations in the traction drive:

– longitudinal body vibrations

$$\left(M_k + 2M_t\right)\ddot{x}_k + \left(4b_b\right)\dot{x}_k - b_b\dot{x}_{kp1} - b_b\dot{x}_{kp2} - b_b\dot{x}_{kp3} - b_b\dot{x}_{kp4} + \left(4c_b\right)x_k - c_bx_{kp1} - c_bx_{kp2} - c_bx_{kp3} - c_bx_{kp4} = F_A - W \tag{38}$$

– vertical body vibrations

$$\begin{gathered} M_k\ddot{z}_k + \left(2b_2 + \left(k_{f1} + k_{f2}\right)b_f\right)\dot{z}_k + \left(-b_2 - k_{f1}b_f\right)\dot{z}_{t1} - \left(k_{f1}b_fL_f\right)\dot{\varphi}_{t1} + \left(-b_2 - k_{f2}b_f\right)\dot{z}_{t2} + \left(k_{f2}b_fL_f\right)\dot{\varphi}_{t2} + \\ + \left(2c_2 + \left(k_{f1} + k_{f2}\right)c_f\right)z_k + \left(-c_2 - k_{f1}c_f\right)z_{t1} - \left(k_{f1}c_fL_f\right)\varphi_{t1} + \left(-c_2 - k_{f2}c_f\right)z_{t2} + \left(k_{f2}c_fL_f\right)\varphi_{t2} = 0 \end{gathered} \tag{39}$$

– body angular vibrations

$$
\begin{gathered}
\left(J_k+2M_tH_k{}^2\right)\ddot{\varphi}_k+\left(2b_2L_k{}^2+b_f(k_{f1}+k_{f2})(L_k+L_f)-4b_bH_k{}^2\right)\dot{\varphi}_k+\left(-b_2L_k-b_fk_{f1}\left(L_k+L_f\right)\right)\dot{z}_{t1}+\\
+\left(-b_fL_f\left(L_k+L_f\right)k_{f1}\right)\dot{\varphi}_{t1}+\left(b_2L_k+b_f\left(L_k+L_f\right)k_{f2}\right)\dot{z}_{t2}+\left(-b_fL_f\left(L_k+L_f\right)k_{f2}\right)\dot{\varphi}_{t2}+\\
+b_bH_k\dot{x}_{kp1}+b_bH_k\dot{x}_{kp2}-b_bH_k\dot{x}_{kp3}-b_bH_k\dot{x}_{kp4}+\left(c_f(L_k+L_f)\left(k_{f1}+k_{f2}\right)\right)z_k+\\
+\left(2c_2L_k^2+4c_bH_k{}^2+c_f(L_k+L_f)^2\left(k_{f1}+k_{f2}\right)\right)\varphi_k+\left(-c_2L_k-c_f(L_k+L_f)k_{f1}\right)z_{t1}+\\
+\left(-c_fL_f(L_k+L_f)k_{f1}\right)\varphi_{t1}+\left(c_2L_k-c_f(L_k+L_f)k_{f2}\right)z_{t2}+\left(-c_fL_f(L_k+L_f)k_{f2}\right)\varphi_{t2}+\\
+c_bH_k(x_{kp1}+x_{kp2}-x_{kp3}-x_{kp4})=F_A\left(H_a-H_s\right)
\end{gathered}
\tag{40}
$$

vertical vibrations of bogies

$$
\begin{gathered}
\left(M_t+2M_d\frac{L_1{}^2}{L_d{}^2}+2\frac{J_d}{L_d^2}\right)\ddot{z}_{t1}+\left(M_d\frac{L_1L_2}{L_d{}^2}-\frac{J_d}{L_d^2}\right)\ddot{z}_{kp1}+\left(M_d\frac{L_1L_2}{L_d{}^2}-\frac{J_d}{L_d^2}\right)\ddot{z}_{kp2}+\left(-b_2-b_fk_{f1}\right)\dot{z}_k+\\
+\left(-b_2L_k-b_f(L_k+L_f)k_{f1}\right)\dot{\varphi}_k+\left(-b_2+b_fk_{f1}+2b_1+2b_{b\varphi}\frac{1}{L_b^2}+2b_d\right)\dot{z}_{t1}+\left(b_fL_fk_{f1}\right)\dot{\varphi}_{t1}+\\
+\left(-b_1-b_{b\varphi}\frac{1}{L_b^2}-b_d\right)\dot{z}_{kp1}+\left(-b_1-b_{b\varphi}\frac{1}{L_b^2}-b_d\right)\dot{z}_{kp2}+\left(-c_2-c_fk_{f1}\right)z_k+\left(-c_2L_k-c_f\left(L_k+L_f\right)k_{f1}\right)\varphi_k+\\
+\left(c_2+2c_1+2c_{b\varphi}\frac{1}{L_b^2}+2c_d+k_{f1}c_f\right)z_{t1}+\left(c_fL_fk_{f1}\right)\varphi_{t1}+\left(-c_1-c_{b\varphi}\frac{1}{L_b^2}-c_d\right)z_{kp1}+\left(-c_1-c_{b\varphi}\frac{1}{L_b^2}-c_d\right)z_{kp2}=-D_1
\end{gathered}
\tag{41}
$$

$$
\begin{gathered}
\left(M_t+2M_d\frac{L_1{}^2}{L_d{}^2}+2\frac{J_d}{L_d^2}\right)\ddot{z}_{t2}+\left(M_d\frac{L_1L_2}{L_d{}^2}-\frac{J_d}{L_d^2}\right)\ddot{z}_{kp3}+\left(M_d\frac{L_1L_2}{L_d{}^2}-\frac{J_d}{L_d^2}\right)\ddot{z}_{kp4}+\left(-b_2-b_fk_{f2}\right)\dot{z}_k+\\
+\left(b_2L_k+b_f(L_k+L_f)k_{f2}\right)\dot{\varphi}_k+\left(b_2+b_fk_{f2}+2b_1+2b_{b\varphi}\frac{1}{L_b^2}+2b_d\right)\dot{z}_{t2}+\left(-b_fk_{f2}L_f\right)\dot{\varphi}_{t2}+\\
+\left(-b_1-b_{b\varphi}\frac{1}{L_b^2}-b_d\right)\dot{z}_{kp3}+\left(-b_1-b_{b\varphi}\frac{1}{L_b^2}-b_d\right)\dot{z}_{kp4}+\left(-c_2-c_fk_{f2}\right)z_k+\left(c_2L_k-c_f\left(L_k+L_f\right)k_{f2}\right)\varphi_k+\\
+\left(c_2+2c_1+2c_{b\varphi}\frac{1}{L_b^2}+2c_d+c_fk_{f2}\right)z_{t2}+\left(c_fL_fk_{f2}\right)\varphi_{t2}+\left(-c_1-c_{b\varphi}\frac{1}{L_b^2}-c_d\right)z_{kp3}+\left(-c_1-c_{b\varphi}\frac{1}{L_b^2}-c_d\right)z_{kp4}=-D_2
\end{gathered}
\tag{42}
$$

– angular vibrations of bogies

$$
\begin{gathered}
J_t\ddot{\varphi}_{t1}+\left(-b_fL_fk_{f1}\right)\dot{z}_k+\left(-b_f(L_k+L_f)L_fk_{f1}\right)\dot{\varphi}_k+\left(b_fL_fk_{f1}\right)\dot{z}_{t1}+\left(b_fL_f{}^2k_{f1}+2(b_1+b_d)L_t{}^2+2b_dL_t{}^2\right)\dot{\varphi}_{t1}+\\
+\left(b_bH_t\right)\dot{x}_{kp1}+\left(-(b_1+b_d)L_t\right)\dot{z}_{kp1}+\left(-b_bH_t\right)\dot{x}_{kp2}+\left((b_1+b_d)L_t\right)\dot{z}_{kp2}+\left(-c_fL_fk_{f1}\right)z_k+\left(-c_f(L_k+L_f)L_fk_{f1}\right)\varphi_k+\\
+\left(c_fL_fk_{f1}\right)z_{t1}+\left(2c_1L_t{}^2-2c_bH_t{}^2+c_fL_f{}^2k_{f1}\right)\varphi_{t1}+\left(c_bH_t\right)x_{kp1}+\left(-c_bH_t\right)x_{kp2}+(-c_1L_t)z_{kp1}+(c_1L_t)z_{kp2}=M_{t1}
\end{gathered}
\tag{43}
$$

$$
\begin{gathered}
J_t\ddot{\varphi}_{t2}+\left(-k_{f2}b_fL_f\right)\dot{z}_k+\left(b_f(L_k+L_f)L_fk_{f2}\right)\dot{\varphi}_k+\left(k_{f2}b_fL_f\right)\dot{z}_{t2}+\left(-b_fL_f{}^2k_{f2}+2\left(b_1+b_d\right)L_t{}^2+2b_dL_t{}^2\right)\dot{\varphi}_{t2}+\\
+\left(b_bH_t\right)\dot{x}_{kp3}+\left(-\left(b_1+b_d\right)L_t\right)\dot{z}_{kp3}+\left(-b_bH_t\right)\dot{x}_{kp4}+\left(\left(b_1+b_d\right)L_t\right)\dot{z}_{kp4}+\left(-c_fL_fk_{f2}\right)z_k+\left(-c_f\left(L_k+L_f\right)L_fk_{f2}\right)\varphi_k+\\
+\left(c_fL_fk_{f2}\right)z_{t2}+\left(2c_1L_t{}^2-2c_bH_t{}^2+c_fL_f{}^2k_{f2}\right)\varphi_{t2}+\left(c_bH_t\right)x_{kp3}+\left(-c_bH_t\right)x_{kp4}+\left(-c_1L_t\right)z_{kp3}+\left(c_1L_t\right)z_{kp4}=M_{t2}
\end{gathered}
\tag{44}
$$

– longitudinal vibrations of wheelsets

$$
\left(M_{kp}+M_d\right)\ddot{x}_{kp1}+\left(-b_b\right)\dot{x}_k-\left(b_bH_k\right)\dot{\varphi}_k-\left(b_bH_t\right)\dot{\varphi}_{t1}+\left(b_b\right)\dot{x}_{kp1}+\left(-c_b\right)x_k+\left(-c_bH_k\right)\varphi_k+\left(-c_bH_t\right)\varphi_{t1}+\left(c_b\right)x_{kp1}=F_{kp1}\tag{45}
$$

$$
\left(M_{kp}+M_d\right)\ddot{x}_{kp2}+\left(-b_b\right)\dot{x}_k-\left(b_bH_k\right)\dot{\varphi}_k+\left(b_bH_t\right)\dot{\varphi}_{t1}+\left(b_b\right)\dot{x}_{kp2}+\left(-c_b\right)x_k-\left(c_bH_k\right)\varphi_k+\left(c_bH_t\right)\varphi_{t1}+\left(c_b\right)x_{kp2}=F_{kp2}\tag{46}
$$

$$
\left(M_{kp}+M_d\right)\ddot{x}_{kp3}+\left(-b_b\right)\dot{x}_k+\left(b_bH_k\right)\dot{\varphi}_k-\left(b_bH_t\right)\dot{\varphi}_{t2}+\left(b_b\right)\dot{x}_{kp3}+\left(-c_b\right)x_k+\left(c_bH_k\right)\varphi_k+\left(-c_bH_t\right)\varphi_{t2}+\left(c_b\right)x_{kp3}=F_{kp3}\tag{47}
$$

$$\left(M_{kp}+M_d\right)\ddot{x}_{kp4}+\left(-b_b\right)\dot{x}_k+\left(b_bH_k\right)\dot{\varphi}_k+\left(b_bH_t\right)\dot{\varphi}_{t2}+\left(b_b\right)\dot{x}_{kp4}+\left(-c_b\right)x_k+\left(c_bH_k\right)\varphi_k+\left(c_bH_t\right)\varphi_{t2}+\left(c_b\right)x_{kp4}=F_{kp4} \quad (48)$$

– vertical vibrations of wheelsets

$$\begin{gathered}\left(M_{kp}+M_d\frac{L_2{}^2}{L_d{}^2}+M_r\right)\ddot{z}_{kp1}+\left(M_d\frac{L_1L_2}{L_d{}^2}-\frac{J_d}{L_d^2}\right)\ddot{z}_{t1}+\left(-b_1-b_{b\varphi}\frac{1}{L_b^2}-b_d\right)\dot{z}_{t1}+\left(-b_1L_t\right)\dot{\varphi}_{t1}+\\+\left(b_1+b_{b\varphi}\frac{1}{L_b^2}+b_d+b_r\right)\dot{z}_{kp1}+\left(-c_1-c_{b\varphi}\frac{1}{L_b^2}-c_d\right)z_{t1}+\left(-c_1L_t\right)\varphi_{t1}+\left(c_1+c_{b\varphi}\frac{1}{L_b^2}+c_d+c_r\right)z_{kp1}=\\=\left(M_r\right)\ddot{\eta}_1+\left(b_r\right)\dot{\eta}_1+\left(c_r\right)\eta_1\end{gathered} \quad (49)$$

$$\begin{gathered}\left(M_{kp}+M_d\frac{L_2{}^2}{L_d{}^2}+\frac{J_d}{L_d^2}+M_r\right)\ddot{z}_{kp2}+\left(M_d\frac{L_1L_2}{L_d{}^2}-\frac{J_d}{L_d^2}\right)\ddot{z}_{t1}+\left(-b_1-b_{b\varphi}\frac{1}{L_b^2}-b_d\right)\dot{z}_{t1}+\left(b_1L_t\right)\dot{\varphi}_{t1}+\\+\left(b_1+b_{b\varphi}\frac{1}{L_b^2}+b_d+b_r\right)\dot{z}_{kp2}+\left(-c_1-c_{b\varphi}\frac{1}{L_b^2}-c_d\right)z_{t1}+\left(c_1L_t\right)\varphi_{t1}+\left(c_1+c_{b\varphi}\frac{1}{L_b^2}+c_d+c_r\right)z_{kp2}=\\=\left(M_r\right)\ddot{\eta}_2+\left(b_r\right)\dot{\eta}_2+\left(c_r\right)\eta_2\end{gathered} \quad (50)$$

$$\begin{gathered}\left(M_{kp}+M_d\frac{L_2{}^2}{L_d{}^2}+\frac{J_d}{L_d{}^2}+M_r\right)\ddot{z}_{kp3}+\left(M_d\frac{L_1L_2}{L_d{}^2}-\frac{J_d}{L_d{}^2}\right)\ddot{z}_{t2}+\left(-b_1-b_{b\varphi}\frac{1}{L_b^2}-b_d\right)\dot{z}_{t2}+\left(-b_1L_t\right)\dot{\varphi}_{t2}+\\+\left(b_1+b_{b\varphi}\frac{1}{L_b^2}+b_d+b_r\right)\dot{z}_{kp3}+\left(-c_1-c_{b\varphi}\frac{1}{L_b^2}-c_d\right)z_{t2}+\left(-c_1L_t\right)\varphi_{t2}+\left(c_1+c_{b\varphi}\frac{1}{L_b^2}+c_d+c_r\right)z_{kp3}=\\=\left(M_r\right)\ddot{\eta}_3+\left(b_r\right)\dot{\eta}_3+\left(c_r\right)\eta_3\end{gathered} \quad (51)$$

$$\begin{gathered}\left(M_{kp}+M_d\frac{L_2{}^2}{L_d{}^2}+\frac{J_d}{L_d^2}+M_r\right)\ddot{z}_{kp4}+\left(M_d\frac{L_1L_2}{L_d{}^2}-\frac{J_d}{L_d^2}\right)\ddot{z}_{t2}+\left(-b_1-b_{b\varphi}\frac{1}{L_b^2}-b_d\right)\dot{z}_{t2}+\left(b_1L_t\right)\dot{\varphi}_{t2}+\\+\left(b_1+b_{b\varphi}\frac{1}{L_b^2}+b_d+b_r\right)\dot{z}_{kp4}+\left(-c_1-c_{b\varphi}\frac{1}{L_b^2}-c_d\right)z_{t2}+\left(c_1L_t\right)\varphi_{t2}+\left(c_1+c_{b\varphi}\frac{1}{L_b^2}+c_d+c_r\right)z_{kp4}=\\=\left(M_r\right)\ddot{\eta}_4+\left(b_r\right)\dot{\eta}_4+\left(c_r\right)\eta_4\end{gathered} \quad (52)$$

– angular vibrations of wheelsets

$$\left(J_{kp}\right)\ddot{\varphi}_{kp1}+\left(b_zR_2{}^2\right)\dot{\varphi}_{kp1}+\left(-b_zR_1R_2\right)\dot{\varphi}_{a1}+\left(c_zR_2{}^2\right)\varphi_{kp1}+\left(-c_zR_1R_2\right)\varphi_{a1}=M_{kp1} \quad (53)$$

$$\left(J_{kp}\right)\ddot{\varphi}_{kp2}+\left(b_zR_2{}^2\right)\dot{\varphi}_{kp2}+\left(-b_zR_1R_2\right)\dot{\varphi}_{a2}+\left(c_zR_2{}^2\right)\varphi_{kp2}+\left(-c_zR_1R_2\right)\varphi_{a2}=M_{kp2} \quad (54)$$

$$\left(J_{kp}\right)\ddot{\varphi}_{kp3}+\left(b_zR_2{}^2\right)\dot{\varphi}_{kp3}+\left(-b_zR_1R_2\right)\dot{\varphi}_{a3}+\left(c_zR_2{}^2\right)\varphi_{kp3}+\left(-c_zR_1R_2\right)\varphi_{a3}=M_{kp3} \quad (55)$$

$$\left(J_{kp}\right)\ddot{\varphi}_{kp4}+\left(b_zR_2{}^2\right)\dot{\varphi}_{kp4}+\left(-b_zR_1R_2\right)\dot{\varphi}_{a4}+\left(c_zR_2{}^2\right)\varphi_{kp4}+\left(-c_zR_1R_2\right)\varphi_{a4}=M_{kp4} \quad (56)$$

– angular vibrations of traction motor armatures

$$\left(J_a\right)\ddot{\varphi}_{a1}+\left(-b_zR_1R_2\right)\dot{\varphi}_{kp1}+\left(b_zR_1{}^2\right)\dot{\varphi}_{a1}+\left(-c_zR_1R_2\right)\varphi_{kp1}+\left(c_zR_1{}^2\right)\varphi_{a1}=M_{a1} \quad (57)$$

$$\left(J_a\right)\ddot{\varphi}_{a2}+\left(-b_zR_1R_2\right)\dot{\varphi}_{kp2}+\left(b_zR_1{}^2\right)\dot{\varphi}_{a2}+\left(-c_zR_1R_2\right)\varphi_{kp2}+\left(c_zR_1{}^2\right)\varphi_{a2}=M_{a2} \quad (58)$$

$$\left(J_a\right)\ddot{\varphi}_{a3}+\left(-b_zR_1R_2\right)\dot{\varphi}_{kp3}+\left(b_zR_1{}^2\right)\dot{\varphi}_{a3}+\left(-c_zR_1R_2\right)\varphi_{kp3}+\left(c_zR_1{}^2\right)\varphi_{a3}=M_{a3} \quad (59)$$

$$\left(J_a\right)\ddot{\varphi}_{a4}+\left(-b_zR_1R_2\right)\dot{\varphi}_{kp4}+\left(b_zR_1{}^2\right)\dot{\varphi}_{a4}+\left(-c_zR_1R_2\right)\varphi_{kp4}+\left(c_zR_1{}^2\right)\varphi_{a4}=M_{a4} \quad (60)$$

Thus, equations (38) - (60) form a simplified mathematical model of the mechanical part of a four-axle section of an electric locomotive. The resulting model is intended to be included in a complex mathematical model of a traction electric drive in order to take into account the mutual influence of electrical processes and mechanical vibrations on each other.

Conclusions. A mathematical model of the mechanical subsystem of the traction electric drive of the four-axle section of the electric locomotive has been developed. A feature of the model is to take into account the interrelated vertical vibrations of the locomotive and torsional vibrations in the traction drive and the influence of the anti-unloading device on the processes in the traction drive. The use of the developed mathematical model of the mechanical subsystem will make it possible to more fully take into account the mutual influence of electrical processes and mechanical vibrations on each other, which will increase the accuracy of modeling. Further improvement of the developed model is possible by introducing nonlinear elastic-dissipative connections between the elements of the mechanical subsystem.

References

1. Postanovlenie KMU ot 03 marta 2021 g. №179 o «Nacional'noj jekonomicheskoj strategii na period do 2030 goda».
2. Spiryagin, M., Wolfs, P., Cole, C., Spiryagin, V., Sun,Y.Q., McSweeney, T. (2016). Design and Simulation of Heavy Haul Locomotives and Trains. Design and Simulation of Heavy Haul Locomotives and Trains. CRC Press. 459. https://doi.org/10.1201/9781315369792.
3. Modelirovanie jelektromehanicheskoj sistemy jelektrovoza s asinhronnym tjagovym dvigatelem [Tekst] / Ju.A. Bahvalov, A.A. Zarif'jan, V.N. Kashnikov [i dr.]; pod red. E.M. Plohova. – M.: Transport, 2001. – 286 s.
4. https://www.3ds.com/products-services/simulia/products/simpack/.
5. http://www.umlab.ru/pages/index.php?id=1#.
6. http://www.gensys.se/.
7. Fedjaeva, G.A. Matematicheskaja model' mehanicheskoj podsistemy tjagovoj jelektroperedachi manevrovovyvoznogo teplovoza TJeM9N / G.A. Fedjaeva, A.N. Tarasov, G.S. Mihal'chenko, N.N. Sidorova // Vestn. Brjan. gos. tehn. un-ta. – 2013.– № 4. – S. 139-142.
8. Rafal's'kij, Oleksandr Oleksandrovich. Pokrashhennja dinamichnih vlastivostej elektroprivodu rudnichnogo elektrovoza v rezhimah buksuvannja [Tekst]: avtoreferat dis … kand. tehn. nauk: 05.22.09 / O.O. Rafal's'kij; Nac. tehn. un-t "Hark. politehn. int". – Harkiv, 2013.
9. Pavlenko A.P. Dinamika tjagovyh privodov magistral'nyh lokomotivov. – M., 1991.
10. Pahomin S.A., Kireev A.V. Pul'sacii momenta tjagovogo reaktivnogo induktornogo dvigatelja v rezhime ogranichenija toka // Izv. vuzov. Jelektromehanika. 2004. – №1. – S. 25-28.
11. Zvancev Pavel Nikolaevich. Issledovanie vertikal'nyh kolebanij, scepnyh i tjagovyh svojstv motornogo vagona jelektropoezda na chetyrjoh odnoosnyh telezhkah: dissertacija...kandidata tehnicheskih nauk: 05.22.07. – Moskva, 2005.
12. Birjukov I.V. Tjagovye peredachi jelektropodvizhnogo sostava zheleznyh dorog. M.: Transport, 1986. – 256 s.
13. V.A. Nehaev, V.A. Nikolaev, M.H. Minzhasarov. Matematicheskoe modelirovanie vertikal'noj dinamiki jelektrovozov novogo pokolenija // Izvestija Transsiba, 2015. – №3(23). – S.31-38.
14. Masliev V.G. Dinamika teplovozov s ustrojstvami, umen'shajushhimi iznos bandazhej koles: monografija / V.G. Masliev; Nac. tehn. un-t "Har'k. politehn. in-t". – H., 2008. – 287 c.
15. Matematicheskoe modelirovanie dinamiki jelektrovozov [Tekst] / A.G. Nikitenko, E.M. Plohov, A.A. Zarif'jan [i dr.]; pod red. A.G. Nikitenko. – M.: Vyssh. shkola, 1998. – 274 s.
16. Elektrovoz VL80s. Rukovodstvo po jekspluatacii. M.; Transport, 1982. 622 s
17. Elektrovoz VL11m: Rukovodstvo po jekspluatacii. – M.: Transport, 1994. 416 s
18. Elektrovoz 2EL5K. Rukovodstvo po jekspluatacii. V pjati knigah. Novocherkassk, 2005.
19. Koropec P.A. Prognozirovanie boksovanija kolesnyh par lokomotiva po harakteristikam dinamicheskih processov v sisteme jekipazh – tjagovyj privod – put'. Avtoref. dis … kand. tehn. nauk. – Rostov n/D, RGUPS, 2007. – 19 s. Special'nost' 05.22.07 – Podvizhnoj sostav zheleznyh dorog, tjaga poezdov i jelektrifikacija.
20. Opir ruhu i kerovanist' rejkovih ekipazhiv [Elektronnij resurs]: monografija /V.P. Tkachenko [ta in.]; Derzhavnij universitet infrastrukturi ta tehnologij. – Kiïv: [b. v.], 2017. – 188 s.
21. Matematicheskoe modelirovanie naprjazhennogo sostojanija povodka podveski tjagovogo jelektrodvigatelja jelektrovoza 2JeS6 / S.G. Shantarenko, V.F. Kuznecov, O.D. Jurasov // Vestnik Rostovskogo gosudarstvennogo universiteta putej soobshhenija: nauchno-tehnicheskij zhurnal. – 2015. – N 2. – S. 46-51
22. Pravila tjagovyh raschetov dlja poezdnoj raboty [Tekst]: utv. M-vom putej soobshhenija SSSR 15.08.80.: – M.: Transport, 1985. – 287 s.
23. V. Tkachenko, S. Sapronova, E. Zub, M. Morneva. (2020). Closed Power Loops in the Guidance of Vehicles by Railway Track System. 24th International Scientific Conference. Transport Means 2020: Sustainability: Research and Solutions (Kaunas, Lithuania). Part II. 554-559. ISSN 1822-296 X (print), ISSN 2351-7034 (on-line).
24. Mikhailov, E., Sapronova, S., Tkachenko, V., Semenov, S., Smyrnova, I., & Kholostenko, Y. (2019). Improved solution of guiding of railway vehicle in curves. In Transport Means – Proceedings of the International Conference, 2019-October 916-921. ISSN 1822-296 X (print); ISSN 2351-7034 (on-line).

25.	Tkachenko, V., & Sapronova, S. (2007). Steerability of railway vehicles. Transport problems, 2(4), 9-16. http://yadda.icm.edu.pl/yadda/element/bwmeta1.element.baztech-article-BSL5-0022-0016
26.	Golubenko, A.L. Sceplenie kolesa s rel'som / A.L. Golubenko – 2-e izd. dop. i per. – Lugansk: Izd-vo VUGU, 1999. – 476 s.

Література

1.	Постановление КМУ от 03 марта 2021 г. №179 о «Национальной экономической стратегии на период до 2030 года».
2.	Spiryagin, M., Wolfs, P., Cole, C., Spiryagin, V., Sun, Y. Q., McSweeney, T. (2016). Design and Simulation of Heavy Haul Locomotives and Trains. Design and Simulation of Heavy Haul Locomotives and Trains. CRC Press. 459. https://doi.org/10.1201/9781315369792
3.	Моделирование электромеханической системы электровоза с асинхронным тяговым двигателем [Текст] / Ю. А. Бахвалов, А. А. Зарифьян, В. Н. Кашников [и др.]; под ред. Е. М. Плохова. – М. : Транспорт, 2001. – 286 с.
4.	https://www.3ds.com/products-services/simulia/products/simpack/
5.	http://www.umlab.ru/pages/index.php?id=1#
6.	http://www.gensys.se/
7. Федяева, Г.А. Математическая модель механической подсистемы тяговой электропередачи маневрововывозного тепловоза ТЭМ9Н/ Г.А. Федяева, А.Н. Тарасов, Г.С. Михальченко, Н.Н. Сидорова// Вестн. Брян. гос. техн. ун-та. - 2013.- № 4. - С. 139-142.
8.	Рафальський, Олександр Олександрович. Покращення динамічних властивостей електроприводу рудничного електровоза в режимах буксування [Текст] : автореферат дис. . канд. техн. наук : 05.22.09 / О. О. Рафальський ; Нац. техн. ун-т "Харк. політехн. інт". - Харків, 2013.
9.	Павленко А.П. Динамика тяговых приводов магистральных локомотивов.-М., 1991.
10.	Пахомин С.А., Киреев А.В. Пульсации момента тягового реактивного индукторного двигателя в режиме ограничения тока // Изв. вузов. Электромеханика. 2004. - №1. - С. 25-28.
11.	Званцев Павел Николаевич. Исследование вертикальных колебаний, сцепных и тяговых свойств моторного вагона электропоезда на четырёх одноосных тележках : диссертация ... кандидата технических наук : 05.22.07. – Москва, 2005.
12.	Бирюков И.В. Тяговые передачи электроподвижного состава железных дорог. М.: Транспорт, 1986. — 256 с.
13.	В. А. Нехаев, В. А. Николаев, М. Х. Минжасаров. Математическое моделирование вертикальной динамики электровозов нового поколения // Известия Транссиба, 2015. – №3(23). – С.31-38.
14.	Маслиев В. Г. Динамика тепловозов с устройствами, уменьшающими износ бандажей колес : монография / В. Г. Маслиев; Нац. техн. ун-т "Харьк. политехн. ин-т". - Х., 2008. - 287 с.
15.	Математическое моделирование динамики электровозов [Текст] / А. Г. Никитенко, Е. М. Плохов, А. А. Зарифьян [и др.]; под ред. А. Г. Никитенко. – М.: Высш. школа, 1998. – 274 с.
16.	Электровоз ВЛ80с. Руководство по эксплуатации. М.; Транспорт, 1982. 622 с
17.	Электровоз ВЛ11м: Руководство по эксплуатации. — М.: Транспорт, 1994. 416 с
18.	Электровоз 2ЭЛ5. Руководство по эксплуатации. В пяти книгах. Новочеркасск, 2005.
19.	Коропец П.А. Прогнозирование боксования колесных пар локомотива по характеристикам динамических процессов в системе экипаж - тяговый привод – путь. Автореф. дис…канд. техн. наук. — Ростов н/Д, РГУПС, 2007. — 19 с. Специальность 05.22.07 - Подвижной состав железных дорог, тяга поездов и электрификация.
20.	Опір руху і керованість рейкових екіпажів [Електронний ресурс] : монографія / В. П. Ткаченко [та ін.] ; Державний університет інфраструктури та технологій. - Київ : [б. в.], 2017. - 188 с.
21.	Математическое моделирование напряженного состояния поводка подвески тягового электродвигателя электровоза 2ЭС6 / С. Г. Шантаренко, В. Ф. Кузнецов, О. Д. Юрасов // Вестник Ростовского государственного университета путей сообщения : научно-технический журнал. - 2015. - N 2. - С. 46-51
22.	Правила тяговых расчетов для поездной работы [Текст]: утв. М-вом путей сообщения СССР 15.08.80.: – М.: Транспорт, 1985. – 287 с.
23.	V. Tkachenko, S. Sapronova, E. Zub, M. Morneva. (2020). Closed Power Loops in the Guidance of Vehicles by Railway Track System. 24th International Scientific Conference. Transport Means 2020: Sustainability: Research and Solutions (Kaunas, Lithuania). Part II. 554-559. ISSN 1822-296 X (print), ISSN 2351-7034 (on-line).
24.	Mikhailov, E., Sapronova, S., Tkachenko, V., Semenov, S., Smyrnova, I., & Kholostenko, Y. (2019). Improved solution of guiding of railway vehicle in curves. In Transport Means - Proceedings of the International Conference, 2019-October 916-921. ISSN 1822-296 X (print); ISSN 2351-7034 (on-line).
25.	Tkachenko, V., & Sapronova, S. (2007). Steerability of railway vehicles. Transport problems, 2(4), 9-16. http://yadda.icm.edu.pl/yadda/element/bwmeta1.element.baztech-article-BSL5-0022-0016
26.	Голубенко, А.Л. Сцепление колеса с рельсом / А.Л. Голубенко – 2-е изд. доп. и пер. – Луганск: Изд-во ВУГУ, 1999. – 476 с.

У статті розроблено математичну модель механічної підсистеми тягового електроприводу чотирьохвісної секції локомотиву. Особливістю моделі є врахування взаємопов'язаних вертикальних коливань локомотива і крутильних коливань у тягової передачі та впливу роботи протирозвантажувального пристрою на процеси в тяговому електроприводі. Це дозволяє врахувати взаємозв'язок коливань екіпажної частини локомотиву, які мають значущий вплив на процеси в електричній частині тягового електроприводу. Модель описує коливання кузова локомотиву, двох візків, чотирьох колісних пар, тягових двигунів та редукторів. При розробці моделі прийнято припущення про лінійну залежність коефіцієнтів жорсткості та демпфування від переміщень та швидкостей. Зчеплення колеса з рейкою описано за допомогою апроксимованої залежності. У модель введено коливання рейок та основи рейкової колії. У якості збурення для вертикальних коливань приймається нерівність рейкової колії. У моделі враховано вплив сили тяги і вертикальних коливань екіпажу на розвантаження осей локомотива. Це дозволить досліджувати процеси при реалізація локомотивом граничних тягових зусиль у тяговому режимі та у режимі електродинамічного гальмування.
Використання розробленої математичної моделі механічної підсистеми дозволить більш повно врахувати взаємний вплив електричних процесів і механічних коливань один на одного, що підвищить точність моделювання. Модель може бути застосовано для досліджень вантажних електровозів типу ВЛ10, ВЛ11, ВЛ82, ВЛ80, 2ЕЛ4, 2ЕЛ5 з метою подальшого удосконалення їх тягових електроприводів, зокрема, визначенню раціональних режимів застосування протирозвантажувального пристрою у тягових та гальмівних режимах. Подальше застосування математичної моделі можливе для оцінки показників тягового електроприводу і локомотиву в цілому при дослідженні сучасних тягових електроприводів, зокрема, асинхронного, застосування яких можливе на вищезазначених локомотивах
Ключові слова: *електровоз, тяговий електропривод, візок, коливання*

В статье разработана математическая модель механической подсистемы тягового электропривода четырехосной секции локомотива. Особенностью модели является учет взаимосвязанных вертикальных колебаний локомотива и крутильных колебаний в тяговой передаче и влияния работы противоразгрузочного устройства на процессы в тяговом электроприводе. Это позволяет учесть взаимосвязь колебаний экипажной части локомотива, которые оказывают значимое влияние на процессы в электрической части тягового электропривода. Модель описывает колебания кузова локомотива, двух тележек, четырех колесных пар, тяговых двигателей и редукторов. При разработке модели принято предположение о линейной зависимости коэффициентов жесткости и демпфирования от перемещений и скоростей. Сцепления колеса с рельсом описано с помощью аппроксимированной зависимости. В модель введено колебания рельсов и основы рельсового пути. В качестве возмущения для вертикальных колебаний принимается неравенство рельсового пути. В модели учтено влияние силы тяги и вертикальных колебаний экипажа на разгрузку осей локомотива. Это позволит исследовать процессы при реализация локомотивом предельных тяговых усилий в тяговом режиме и в режиме электродинамического торможения.
Использование разработанной математической модели механической подсистемы позволит более полно учесть взаимное влияние электрических процессов и механических колебаний друг на друга, что повысит точность моделирования. Модель может быть применено для исследований грузовых электровозов типа ВЛ10, ВЛ11, ВЛ82, ВЛ80, 2ЕЛ4, 2ЕЛ5 с целью дальнейшего совершенствования их тяговых электроприводов, в частности, определению рациональных режимов применения противоразгрузочного устройства в тяговых и тормозных режимах. Дальнейшее применение математической модели возможно для оценки показателей тягового электропривода и локомотива в целом при исследовании современных тяговых электроприводов, в частности, асинхронного, применение которых возможно на вышеупомянутых локомотивах.
Ключовые слова: *электровоз, тяговый электропривод, тележка, колебания*

Гулак С.О. – доцент кафедри електромеханіки та рухомого складу залізниць Державного університету інфраструктури та технологій, кандидат технічних наук,
ORCID ID: 0000-0002-2294-5676
Сапронова С.Ю. – професор кафедри вагони та вагонне господарство Державного університету інфраструктури та технологій, професор, доктор технічних наук,
ORCID ID: 0000-0002-1482-1665
Ткаченко В.П. – завідувач кафедри електромеханіки та рухомого складу залізниць Державного університету інфраструктури та технологій, професор, доктор технічних наук,
ORCID ID: 0000-0002-5513-2436
Рябов Є.С. – доцент кафедри електричного транспорту та тепловозобудування Національного технічного університету «Харківський політехнічний інститут», старший науковий співробітник, кандидат технічних наук,
ORCID ID: 0000-0003-0753-514X
Овер'янова Л.В. – доцент кафедри електричного транспорту та тепловозобудування Національного технічного університету «Харківський політехнічний інститут», доцент, кандидат технічних наук,
ORCID ID**:** 0000-0002-4827-572X
Єріцян Б.Х. – доцент кафедри електричного транспорту та тепловозобудування Національного технічного університету «Харківський політехнічний інститут», доцент, кандидат технічних наук,
ORCID ID: 0000-0003-0579-3882